Directional Control in Light-Wave Guidance

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The transmission of light waves for communication in a medium sheltered from atmospheric effects requires wave guidance providing frequent changes in direction of propagation. This paper shows that, in any electromagnetic waveguide having transverse planes in which the field is essentially equiphase, the transverse width of the field distribution 2a and wavelength λ *determine the order of magnitude of the direction-determining parameters,* R_{min}, *the minimum bending radius, and* δ_{max}, *the maximum abrupt angular changes, according to the relations*

$$R_{min} = 2(a^3/\lambda^2)$$

$$\delta_{max} = \tfrac{1}{2}(\lambda/a)$$

which are valid in the region $\lambda < a$. *The significance of* R_{min} *is apparent, with the note that in a system containing a multiplicity of bends, an appropriate way of summing the effects of the individual bends should be used to establish an over-all equivalent bend radius for the complete transmission path, which must be larger than* R_{min}. *The quantity* δ_{max} *may be regarded as the maximum value of the accumulated angular errors (rms sum, for example) in a transmission line including reflecting or refracting elements for directional control. For a light beam at* $\lambda = 0.6328$ *microns having a diameter of 1.0 mm,* $\delta_{max} = 0.036°$ *and* $R_{min} = 600$ *meters.*

Small-diameter beams ease the problem of directional control. There is no fundamental reason why small beams should not be achievable with low loss in the straight condition, but many guiding structures do have an inverse relation between beam diameter and straight-condition attenuation coefficient. To explore the direction-controlling properties of specific media and the interaction of R_{min} *and* δ_{max} *with straight attenuation coefficient, the following waveguides and associated criteria for establishing* R_{min} *and* δ_{max} *were studied:*

(1) sequence of lenses: criterion, beam deflection from nominal axis by one beam radius,

(2) hollow dielectric waveguide: criterion, added bend loss equal to straight condition loss,

(3) round metallic circular-electric waveguides: for helix guide, criterion is bend loss equal to straight loss; for simple metallic tube, criterion is a transmission ripple (due to mode conversion) of about 1.7 db.

In all cases the functional dependence on a and λ for R_{min} or δ_{max} was the same (given above) as derived for the generalized electromagnetic waveguide, and the associated constants were in most cases of similar magnitude.

I. INTRODUCTION

In research on techniques for transmitting light waves over appreciable distances for communication it has become evident that control of direction of propagation is an important and difficult problem. Electromagnetic waves in free space travel in a straight line. In a medium that is sheltered from atmospheric effects, frequent changes in direction are necessary to follow vertical terrain contours and to conform to a horizontal path avoiding physical obstacles and regions of high-cost installation. The wave guiding medium must provide these direction changes.

In this paper some simple relations are derived to give the order of magnitude of the direction-determining factors, bending radius and abrupt tilt angle, for any wave guiding structure as a function of wavelength and the transverse dimension of the guided electromagnetic wave beam. These simple relations are then compared to the corresponding more precisely defined quantities for specific waveguides: (1) a sequence of lenses,[1] (2) the hollow-dielectric waveguide,[5] and (3) round waveguides for circular electric waves.

II. DERIVATION OF GENERAL WAVEGUIDE DIRECTIONAL SENSITIVITY

In Fig. 1 we show a generalized waveguide for electromagnetic waves, with an abrupt open end radiating into free space. We assume the field at the aperture is essentially equiphase, which implies ending the guide

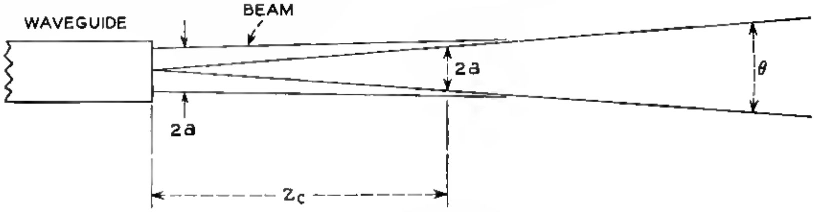

Fig. 1 — Waveguide with abrupt open end.

only at certain longitudinal locations if a periodic form of guidance (such as a sequence of lenses) is employed. Let the field strength variation across the aperture be approximately sinusoidal. Then, approximately, the far-field beam angle θ is

$$\theta = \lambda/a \text{ radians} \tag{1}$$

in which we require $a > \lambda$. Other aperture distributions would give the same order of magnitude for θ. In the near-field region the radiated beam remains collimated in a width approximately $2a$ out to a distance z_c from the aperture, where

$$z_c\theta = 2a \tag{2}$$

$$z_c = 2a^2/\lambda. \tag{2a}$$

The key inference on directional sensitivity is introduced here. Since in the absence of the guide the beam remains confined to essentially the same region as in the presence of the guide, it is concluded that the guide has little influence on the beam over the interval z_c. Thus any appreciable change in direction of wave propagation must not be made in a distance less than z_c.

With reference to Fig. 2, the departure of a circular arc from the tangent is

$$\Delta = \tfrac{1}{2}(l^2/R). \tag{3}$$

We now require that $\Delta = a$ when $l = z_c$. Using (3), (2) and (1), we obtain the minimum bend radius $R_{\min}$

$$R_{\min} = 2a^3/\lambda^2. \tag{4}$$

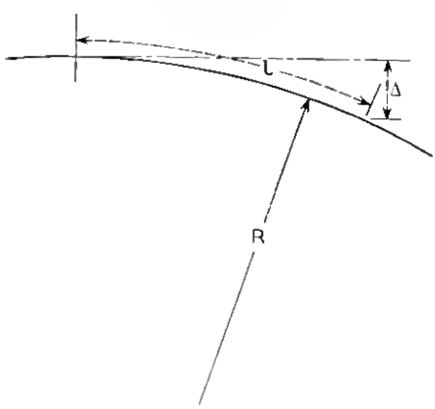

Fig. 2 — Departure of a circular arc from the tangent.

Alternatively, this same relation may be arrived at by specifying that the change in direction shall be one beamwidth θ after traveling a distance z_c in the minimum bending radius $R_{\min}$; i.e., $R_{\min}\theta = z_c$.

Equation (4) gives the order of magnitude of bend radius at which the wave propagation will change character. At longer bend radii the wave propagation will be essentially as in the straight guide, and at shorter bend radii something drastic will happen. Just what changes occur in the latter case depend on the nature of the medium in detail. If the medium is enclosed in a perfect conductor the change will be large mode conversion. If it consists of a sequence of infinitely wide lenses we will see that the change is a wide oscillation of the beam about the nominal axis of propagation. Note that in neither of these cases is energy lost due to the bend. Nonetheless, we regard either change as undesirable.

Consider an abrupt angular change in the guide direction, δ. Following a line of reasoning analogous to that given above, we can say that the character of wave propagation will change rapidly in the region where

$$\delta_{\max} = \theta/2 = \lambda/2a. \tag{5}$$

Smaller values of δ will cause progressively less change in wave character, whereas larger values of δ will cause violent changes.

If we consider the relation of these quantities to a wave guiding medium, it is apparent that $R_{\min}$ is intended as the smallest radius at which the otherwise uniform medium can be bent. When a multiplicity of bends is included in a single transmission link, some way of summing their effects is needed to form an equivalent bending radius which must be greater than $R_{\min}$.

The angle δ is somewhat different. In many media where $a \gg \lambda$ it is possible to insert a large plane reflector and introduce a change of direction of arbitrary size. As long as the guides at both approaches to the reflector are perfectly aligned according to geometric optics, the disturbance on wave propagation may be negligible. However, there will be an error in such angular alignment and $\delta_{\max}$ tells us how large that error may be. When many random angular errors are made, $\delta_{\max}$ is approximately the rms accumulation of such errors.

The numerical values of $R_{\min}$ and $\delta_{\max}$ have been plotted for λ from 0.6328 to 10 microns and beam radius a from 0.1 to 100 millimeters in Figs. 3 and 4, respectively. For example, at $\lambda = 0.6328$ microns and $2a = 1.0$ cm, $R_{\min} = 600{,}000$ meters and $\delta_{\max} = 3.6 \times 10^{-3}$ degrees. Dropping to $2a = 1.0$ mm, $R_{\min} = 600$ meters and $\delta_{\max} = 3.6 \times 10^{-2}$ degrees.

We consider next certain specific wave guiding structures to compare

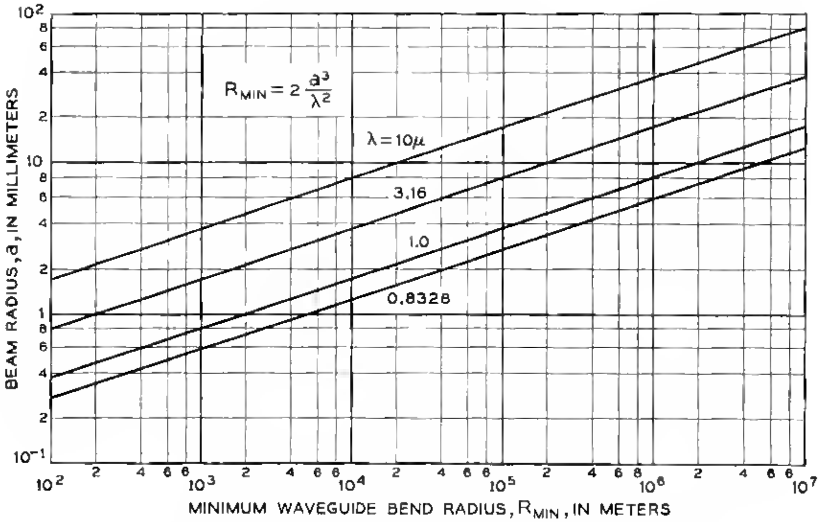

Fig. 3 — Beam radius vs minimum waveguide bend radius.

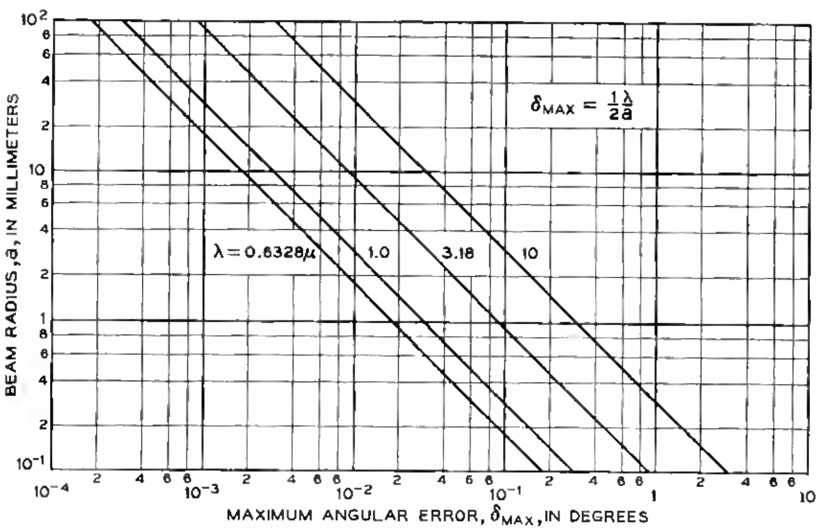

Fig. 4 — Beam radius vs maximum angular error.

the results of directional changes in those structures to the generalized conclusions drawn above.

III. SEQUENCE-OF-LENS WAVEGUIDE

G. Goubau has proposed[1] a waveguide for electromagnetic waves consisting of a series of lenses, and D. Marcuse has used geometric optics to determine the effects of bends in such a waveguide.[2]

If the input to a lens waveguide is a ray which is inclined at an angle δ to the longitudinal waveguide axis (Fig. 5) the departure of the ray from the longitudinal axis has a magnitude at successive lenses which is contained within an envelope which is a sinusoidal function of distance along the longitudinal axis. Starting with the work of Marcuse, one can show that there is an optimum strength of lens which minimizes the departure of a ray from the axis; the optimum focal length f is related to the lens spacing L by

$$2f = L \tag{6}$$

and under that condition the maximum deviation of the ray from the longitudinal axis is

$$r_{\max-1} = \delta L. \tag{7}$$

Consider a region of bend radius R following a straight region of lens waveguide. For a ray incident on the curved region from the axis of the straight region Marcuse has also calculated the ray's departure from the axis in the curved region; for the case $f = L/2$ the maximum departure is

$$r_{\max-2} = L^2/R. \tag{8}$$

We now relate these departures from the guide axis to the transverse dimension of the beam. It is convenient to consider the beam radius to be that value of radius beyond which a completely negligible amount of

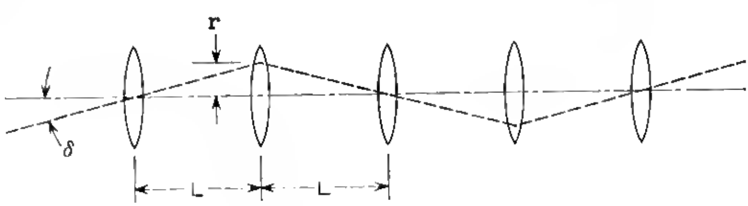

Fig. 5 — Sequence of lenses.

field exists. We define this beam radius as

$$r_b = (N_0 L \lambda)^{\frac{1}{2}}. \tag{9}$$

Here N_0 is the Fresnel number which previous work[3,9] shows is on the order of unity for negligibly small diffraction loss when all energy outside the radius r_b is absorbed at the lenses.*

As a criterion of the maximum permissible abrupt angular change, we somewhat arbitrarily set it to be that angle at which $r_{\max-1}$ is equal to the transverse beam radius r_b :

$$\delta_{\max} = (r_b/L) = (N_0\lambda/L)^{\frac{1}{2}} = N_0\lambda/r_b . \tag{10}$$

Since $N_0 \cong 1$, this specific guide and criterion gives a permissible angular change of twice that prescribed by (5). This may be considered an excellent agreement.

As a criterion of the minimum permissible bend radius, we set the resulting beam deflection $r_{\max-2}$ equal to the transverse beam radius r_b :

$$R_{\min} = L^2/r_b = r_b^3/(N_0\lambda)^2. \tag{11}$$

Since $N_0 \cong 1$, this specific guide and criterion gives a permissible bend radius of one-half that prescribed by (4), which again may be considered excellent agreement.

The most important aspect of the comparison between (4) and (5), (10), and (11) is that the corresponding equations have the identical dependence on λ and a, which determines in a broad way the magnitude of the direction determining parameters.

In this form of guide we can readily relate the beam radius to the associated lens spacing and the losses. Fig. 6 shows the lens spacing L versus beam radius in the 0.5- to 4-micron wavelength region. As before, N_0 will be about unity, but where extremely low losses per lens are required may have to be slightly greater than unity.[3] For the 1.0-mm beam diameter referred to above, the lens spacing is about 0.4 meter for $\lambda = 0.63$ microns.

In principle, vanishingly small transmission loss could be obtained by appropriate choice of lens diameter (i.e., choice of N_0), if the reflection, absorption, and scattering losses were negligible at the lenses. In practice, such losses may be very real. Fig. 7 shows the total losses per lens required as a function of lens spacing with net transmission loss as a parameter. For 3 db/mile net loss and the 0.4-meter lens spacing, a power loss per lens of about one part in 10^4 is required.

* Further discussion of N_0 and r_b is given in the Appendix.

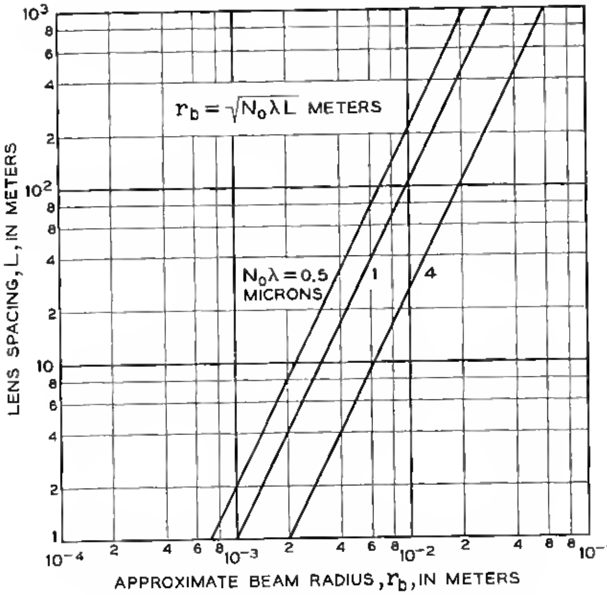

Fig 6. — Approximate beam radius vs lens spacing in a sequence of identical lenses.

In general, of course, the smaller beam diameters which permit rapid direction changes require more tight guidance (closer lens spacing) and tend to increase the losses. Note, however, that the only inherent losses associated with tight guidance for the lens guidance system are due to scattering or reflection at lens surfaces or bulk lens absorption loss, both of which may conceivably be made very small.

IV. HOLLOW DIELECTRIC WAVEGUIDE

E. A. J. Marcatili and R. A. Schmeltzer have proposed a waveguide for light waves consisting of a hollow dielectric tube in which the useful energy is entirely confined to the central hole.[5] When the guide is straight, loss takes place through very slow radiation into the dielectric, which is completely absorbing for the light energy.

The bending radius for such a guide which makes the extra loss due to bending (also a radiation loss) exactly equal to the straight-guide attenuation coefficient has also been determined.[5] They find, for the lowest-order mode (EH_{11}), in which the field varies roughly cosinusoidally from the axis to the inner wall of the tube,

$$R_{\min-D} = 9.5\, a^3/\lambda^2 \tag{12}$$

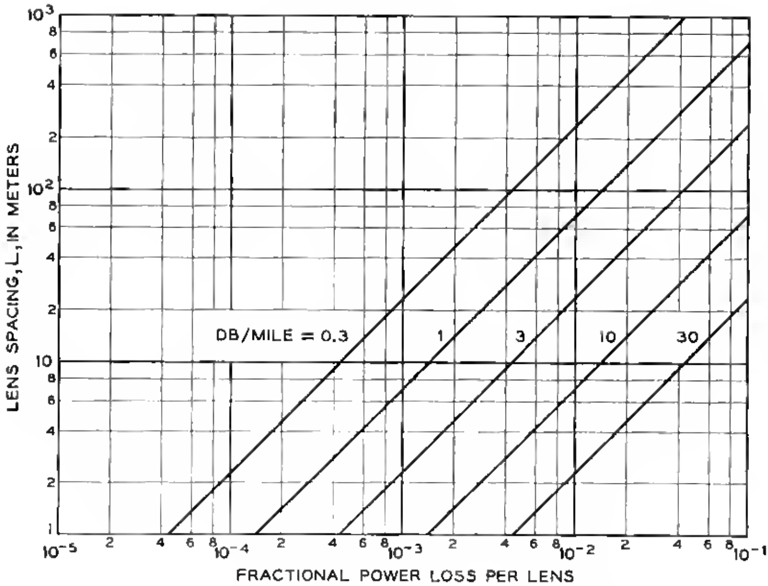

Fig. 7 — Lens loss vs spacing for prescribed total loss.

where a is the inner radius of the tube. Once again, the functional dependence of $R_{\min-D}$ on a and λ is identical to that in (4).

The straight-line attenuation coefficient for the lowest-order mode can be reduced to[5]

$$\alpha_s = 0.214\ \lambda^2/a^3 \tag{13}$$

for an index of refraction of the dielectric tube equal to 1.5. As Marcatili and Schmeltzer have pointed out, the dependence of α_s on a and λ is the exact inverse of that for $R_{\min-D}$; hence for any prescribed straight-guide loss there is a minimum permissible bending radius, which for the lowest-order mode is

$$R_{\min-D} = 2.03/\alpha_s\ . \tag{14}$$

For fixed α_s this is independent of λ. For $R_{\min-D} = 1000$ meters, $\alpha_s = 0.002$ nepers/meter or 27.9 db/mile, and at $\lambda = 0.6328$ microns, the inner radius of the tube $a = 0.35$ mm.

V. CIRCULAR ELECTRIC WAVEGUIDES

We consider now the directional control relations for round waveguides designed for the TE_{01} circular electric wave. Helix waveguide and smooth-

walled metallic waveguide involve quite different criteria for tolerable bending radius and will be discussed separately.

Consider first metallic guides in which the losses are negligible compared to the mode coupling coefficients. We assume the degeneracy between TE_{01} and TM_{11} is broken with a dielectric lining or other guide modification. Then the limit on bending radius or abrupt tilt is the interfering effect between the unperturbed TE_{01} energy and the energy which is converted to an undesired mode and reconverted back to the TE_{01} wave.

For an abrupt tilt, the amplitude conversion coefficient from TE_{01} to TE_{12} was found by S. P. Morgan[4] to be, approximately

$$p = 1.935\ (a/\lambda)\delta \tag{15}$$

where δ is the tilt angle in radians. When converted energy from one tilt strikes another tilt (presumed for simplicity here to be the same angle), energy is reconverted back to the TE_{01} wave with the same conversion coefficient given by (15). The amplitude of the reconverted wave compared to the unperturbed wave is then p^2. Depending on the relative phase of the reconverted vector, it may add at any phase angle to the unperturbed wave. Hence the amplitude transmission coefficient varies with wavelength between $(1 + p^2)$ and $(1 - p^2)$. Letting a transmission fluctuation of 1.7 db, corresponding to $p^2 = 0.1$, be the criterion of limiting tilt angle, we find

$$\delta_{\max - M} = p\lambda/1.935a = 0.164\ \lambda/a. \tag{16}$$

Comparing this to the generalized relation for $\delta_{\max}$ in (5), we note the identical dependence on λ and a and a somewhat smaller constant multiplier. In practice, the existence of coupling to other modes would tend to make somewhat smaller values of $\delta_{\max - M}$ needed, but the dependence on λ and a would not be affected.

Still considering guides with negligible losses, we examine the effect of a constant-radius bend. It may be shown that the reconverted vector has the magnitude

$$p_1^2 = k_t^2/(\Gamma_1 - \Gamma_2)^2 \tag{17}$$

where k_t is the distributed coupling coefficient between TE_{01} and an undesired mode and Γ_1 and Γ_2 are the propagation constants for TE_{01} and the undesired modes, respectively. For TE_{01} to TE_{12}

$$k_t \cong j(2a/\lambda_0 R) \tag{18}$$

where R is the bend radius and a is the radius of the guide. Also,

$$(\Gamma_1 - \Gamma_2) \cong j(\beta_1\beta_2) = j\ \lambda_0/a^2. \tag{19}$$

Hence

$$p_1^2 = 4a^6/R^2\lambda^4. \tag{20}$$

As a criterion for minimum bend radius we set p_1^2 equal to 0.1, giving the same ripple in transmission loss as noted above, with the resulting bending radius from (20)

$$R_{\min-M} = (4/p^2)^{\frac{1}{2}}(a^3/\lambda^2) = 6.3\ a^3/\lambda^2. \tag{21}$$

Comparing (21) to (4), we again find the identical dependence on a and λ with a slightly different constant.

Turning now to the case of helix waveguide in which very strong loss is introduced for the undesired mode, we find we do not have explicit forms for the coupling coefficient. We take advantage of some numerical evaluations carried out in the 30- to 100-kmc region on guide varying in diameter from 0.25 inch to 3 inches. It was found that the bend loss coefficient is given by the expression*

$$\alpha_B = 0.0726\ (a^3/R^2\lambda^{2.7}). \tag{22}$$

The TE_{01} loss of the guide when straight is very nearly that of a copper cylinder, given by S. A. Schelkunoff as[6]

$$\alpha_s = 4.46 \times 10^{-6}\ \lambda^{\frac{1}{2}}/a^3. \tag{23}$$

In (23) we have assumed $a^2 \gg (\lambda/2)^2$, so that the cutoff effect is negligible. As our criterion for minimum bending radius we equate the bend loss α_B and the straight-line loss α_s, yielding

$$R_{\min-H} = 128\ a^3/\lambda^{2.1}. \tag{24}$$

The functional dependence of $R_{\min-H}$ on λ and a is very nearly the same as in (4), but the constant multiplier is much greater. This is a consequence of the criterion $\alpha_s = \alpha_B$, which is thereby proven much more stringent than the rather lax transmission ripple criterion used above for the metallic tube guide in which dissipation was negligible. Since α_s of (23) and $R_{\min-H}$ of (24) have different dependence on λ, a change of wavelength will influence $R_{\min-H}$ even though α_s is held constant. We can express this by substituting (23) into (24), giving

$$R_{\min-H} = \frac{128}{\lambda^{0.6}} \times \frac{a^3}{\lambda^{\frac{3}{2}}} = \frac{5.71 \times 10^{-4}}{\lambda^{0.6}\alpha_s}. \tag{25}$$

At longer wavelengths, *smaller* bending radii are tolerable even though

* This is the result of unpublished calculations by the author, based on coupling coefficients derived by methods due to Unger[9] and using coupled-wave theory.[10]

α_s is held constant by increasing a. At a wavelength of 5 mm and a straight attenuation coefficient of 1 db/mile, $R_{\min-H} = 191$ meters. This result and the numerical constant in (22) are dependent to some extent on the wall impedance to the undesired modes used in the numerical evaluations referred to above, which was on the order of one-half the free-space intrinsic impedance.

VI. CONCLUSION

For any guided electromagnetic wave, the order of magnitude of the direction-determining parameters $R_{\min}$ (the minimum bending radius) and $\delta_{\max}$ (the maximum abrupt angular change) are uniquely determined by the wavelength and transverse beam dimension. Equations (4) and (5) were derived, determining $R_{\min}$ and $\delta_{\max}$ for a general guided electromagnetic wave by inferring the tightness of guidance from the behavior of a wave radiated from the open end of the waveguide. Investigation of specific forms of waveguide with precise criteria for setting limits on R and δ (as outlined in the abstract) lead to identical functional forms for $R_{\min}$ and $\delta_{\max}$, with similar constant multipliers.

APPENDIX

Previous workers[3,7] have calculated the diffraction loss at a reflector in a maser interferometer, and the same loss per lens would be expected in a sequence-of-lens waveguide if the entire plane outside the edge of the lens (of radius equal to that of the maser reflector) were absorbing. These losses are plotted versus $N = a^2/L\lambda$ (where a is the reflector radius) in Fig. 3 of Ref. 7 and in Fig. 15 of Ref. 3 for focal length $f = L/2$. We chose N_0 to be that value of N which gives satisfactorily low loss per lens; for example, for $N_0 = 1$, Ref. 7 gives a power loss per lens of one part in 10^4 for the lowest-order wave, and for $N_0 \cong 1.4$ the power loss is one part in 10^6. Fig. 3 of Ref. 1 shows that 99.8 per cent of the energy of the normal mode for infinite lenses lies within the radius $r = (L\lambda)^{\frac{1}{2}}$ at the lens. In practical cases, therefore, N_0 will differ little from unity.

Another item of interest is the relation between r_b of (9) and the field amplitude given by previous workers.[7,8] The field varies as a function of radius r from the axis of the guide according to

$$\exp\left(-r^2/w^2\right) \tag{26}$$

where w is the radius at which the field drops to e^{-1} of its maximum (on axis) value. The value of w varies with longitudinal position between

lenses; at midway between lenses $w = w_0$, where

$$w_0 = (L\lambda/2\pi)^{\frac{1}{2}}. \tag{27}$$

At the lenses, $w = w_s$, and for our cases of $f = L/2$

$$w_s = (L\lambda/\pi)^{\frac{1}{2}}. \tag{28}$$

It is apparent from (9) and (28) that

$$r_b = w_s\,(N_0\pi)^{\frac{1}{2}}. \tag{29}$$

In terms of w_s (10) becomes

$$\delta_{\max} = (N_0/\pi)^{\frac{1}{2}}(\lambda/w_s) \tag{30}$$

and (11) becomes

$$R_{\min} = (\pi^{\frac{1}{2}}N_0^{-\frac{1}{2}})\,(w_s^{\,3}/\lambda^2). \tag{31}$$

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